



Metoda elementów skończonych (MES1)

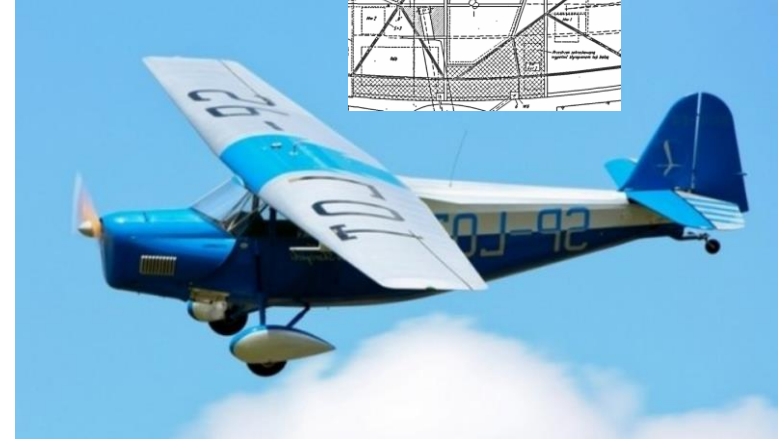
Wykład 7C. 2D element pręta kratownicy

04.2022

Przykłady kratownic



Kratownica mostu



Kratownica kadłuba

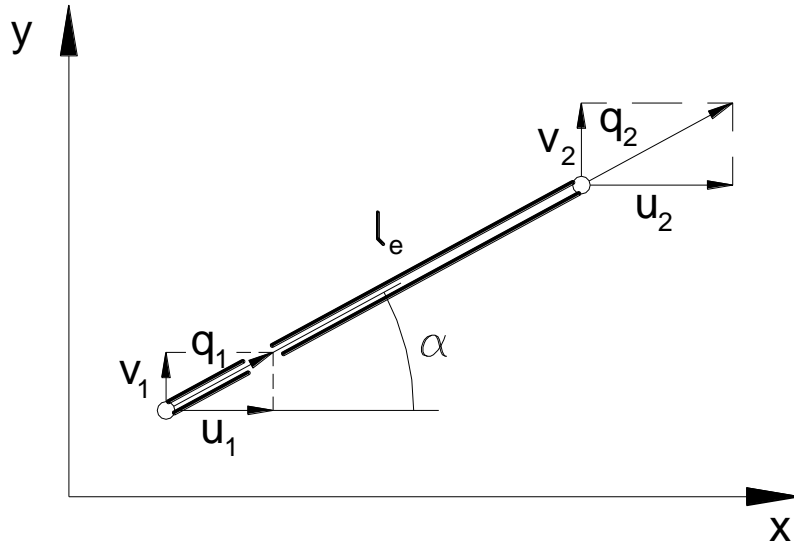


Kratownica dźwigu



Kratownica dachu

Element skończony pręta kratownicy 2D



Lokalny wektor parametrów węzłowych: $\{q\}_e = \{q_1, q_2\}_e$

Globalny wektor parametrów węzłowych:

$$\{q_g\}_e = \{u_1, v_1, u_2, v_2\}_e$$

Transformacja wektora parametrów węzłowych:

$$q_i = u_i \cos \alpha + v_1 \sin \alpha \quad (i=1,2)$$

$$\{q\}_e = [T_k] \{q_q\}_e$$

Lokalna macierz sztywności pręta: $[k]_e = \frac{EA}{l_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

Energia sprężysta elementu:

$$U_e = \frac{1}{2} \underbrace{\{q\}_e^T}_{1 \times 2} \underbrace{[k]_e}_{2 \times 2} \underbrace{\{q\}_e}_{2 \times 1} = \frac{1}{2} \underbrace{\{q_q\}_e^T}_{1 \times 4} \underbrace{[T_k]^T}_{4 \times 2} \underbrace{[k]_e}_{2 \times 2} \underbrace{[T_k]}_{2 \times 4} \underbrace{\{q_q\}_e}_{4 \times 1}$$

$$U_e = \frac{1}{2} \{q_q\}_e^T [k_g]_e \{q_q\}_e$$

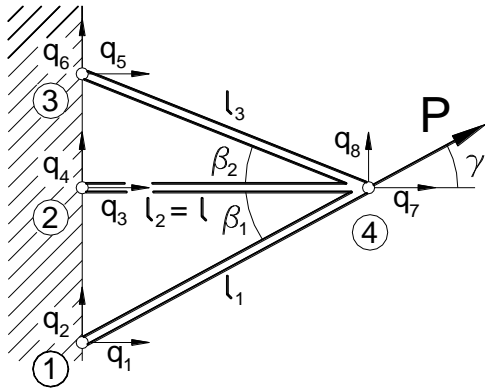
$$\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_e = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 & 0 \\ 0 & 0 & \cos \alpha & \sin \alpha \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}_e$$

Globalna macierz sztywności pręta kratownicy:

$$[k_g]_e = \frac{EA}{l_e} \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

$s = \sin \alpha, c = \cos \alpha$ **3**

Przykład kratownica 3 prętowa



$$[k_{ij}]_e^1, [k_{ij}]_e^2, [k_{ij}]_e^3$$

- Element 1 nodes 1 and 4 slope angle $\alpha_1 = \beta_1$ length $l_1 = \frac{l}{\cos \alpha_1}$.
- Element 2 nodes 2 and 4 slope angle $\alpha_2 = 0$ length $l_2 = \frac{l}{\cos \alpha_2}$.
- Element 3 nodes 3 and 4 slope angle $\alpha_3 = -\beta_2$ length $l_3 = \frac{l}{\cos \alpha_3}$.

k_{11}^1	k_{12}^1	0	0	0	0	k_{13}^1	k_{14}^1	$\left\{ \begin{array}{l} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ q_7 \\ q_8 \end{array} \right\} = \left\{ \begin{array}{l} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ P \cos \gamma \\ P \sin \gamma \end{array} \right\}$
k_{21}^1	k_{22}^1	0	0	0	0	k_{23}^1	k_{24}^1	
0	0	k_{11}^2	k_{12}^2	0	0	k_{13}^2	k_{14}^2	
0	0	k_{21}^2	k_{22}^2	0	0	k_{23}^2	k_{24}^2	
0	0	0	0	k_{11}^3	k_{12}^3	k_{13}^3	k_{14}^3	
0	0	0	0	k_{21}^3	k_{22}^3	k_{23}^3	k_{24}^3	
k_{31}^1	k_{32}^1	k_{31}^2	k_{32}^2	k_{31}^3	k_{32}^3	$k_{33}^1 + k_{33}^2 + k_{33}^3$	$k_{34}^1 + k_{34}^2 + k_{34}^3$	
k_{41}^1	k_{42}^1	k_{41}^2	k_{42}^2	k_{41}^3	k_{42}^3	$k_{43}^1 + k_{43}^2 + k_{43}^3$	$k_{44}^1 + k_{44}^2 + k_{44}^3$	

Warunki brzegowe:

$$q_j = 0 \quad j = 1, 6$$

$$EA \begin{bmatrix} \sum_{i=1}^3 \frac{c_i^2}{l_i} & \sum_{i=1}^3 \frac{s_i c_i}{l_i} \\ \sum_{i=1}^3 \frac{s_i c_i}{l_i} & \sum_{i=1}^3 \frac{s_i^2}{l_i} \end{bmatrix} \begin{Bmatrix} q_7 \\ q_8 \end{Bmatrix} = \begin{Bmatrix} P \sin \gamma \\ P \cos \gamma \end{Bmatrix}$$

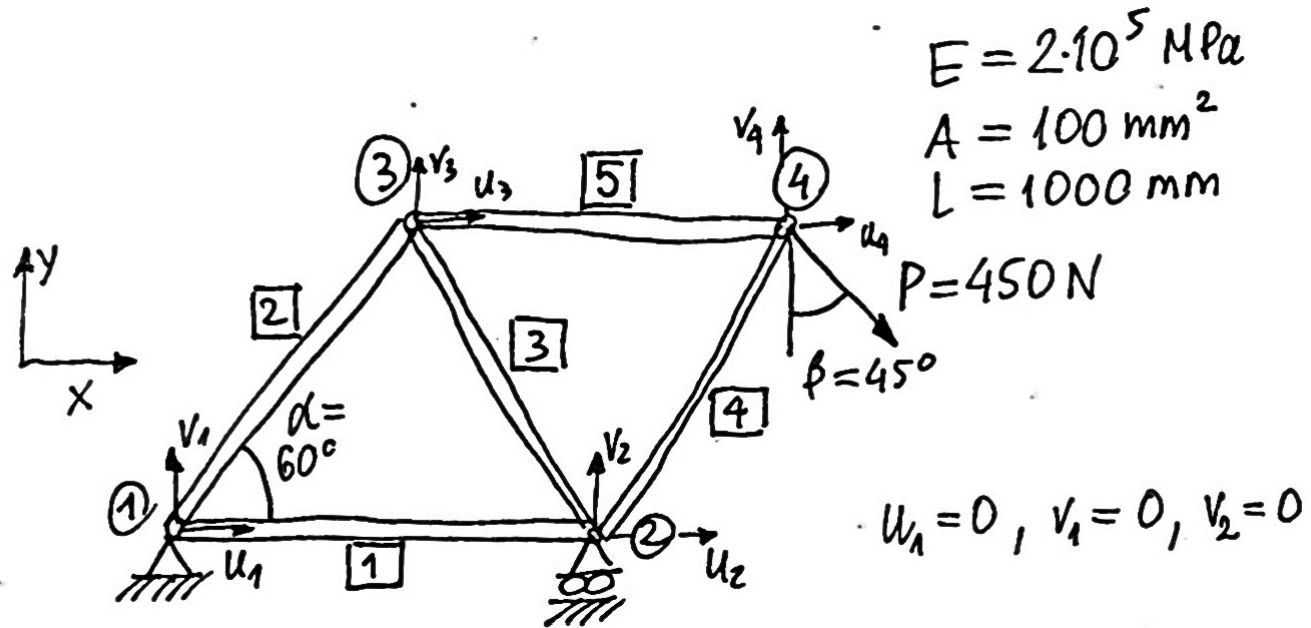
Dla: $\beta_1 = \beta_2 = \beta \quad \gamma = 0$

$$\frac{EA}{l} \begin{bmatrix} 1 + 2c^3 & 0 \\ 0 & 2s^2 c \end{bmatrix} \begin{Bmatrix} q_7 \\ q_8 \end{Bmatrix} = \begin{Bmatrix} P \\ 0 \end{Bmatrix}$$

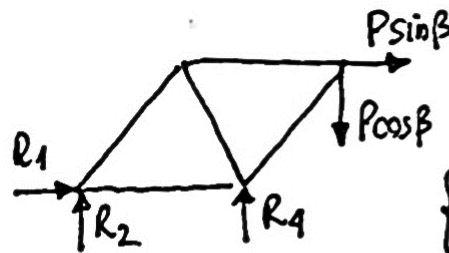
Dla: $c = \cos \beta$
 $s = \sin \beta$

$$q_7 = \frac{Pl}{EA(1 + 2c^3)} \quad q_8 = 0$$

Przykład Zbuduj model MES 2 wymiarowej kratownicy. Znajdź przemieszczenia węzłowe, naprężenia, siły wewnętrzne i reakcje



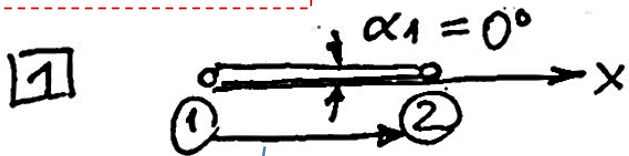
$$\{q\}_{8 \times 1} = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$



$$\{F\}_{8 \times 1} = \begin{Bmatrix} R_1 \\ R_2 \\ 0 \\ R_4 \\ 0 \\ 0 \\ P \sin \beta \\ -P \cos \beta \end{Bmatrix}$$

$$[k_s]_e = \frac{EA}{l_e} \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

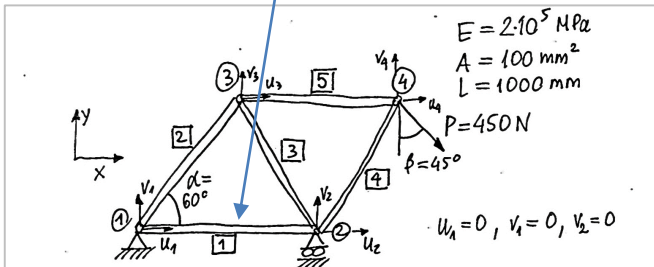
$s = \sin \alpha, c = \cos \alpha$



$$c_1 = 1, s_1 = 0, [q_g]_1 = [u_1, v_1, u_2, v_2]_{1 \times 4}$$

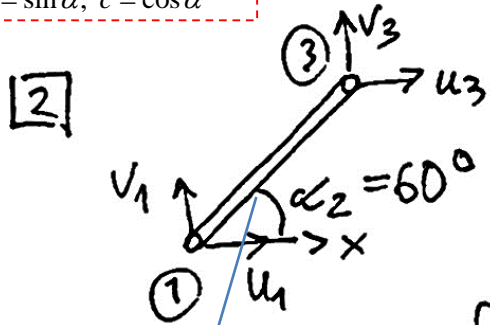
$$[k_g]_1 = \frac{EA}{l} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \frac{EA}{4l} \begin{bmatrix} 4 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 \\ -4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[[k_g]_1]^* = \frac{EA}{4l} \begin{bmatrix} 4 & 0 & -4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -4 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



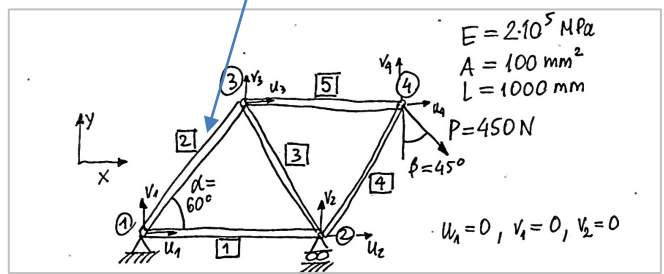
$$[k_s]_e = \frac{EA}{l_e} \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

$s = \sin \alpha, c = \cos \alpha$



$$c_2 = \frac{1}{2}, s_2 = \frac{\sqrt{3}}{2}, [q/g]_2 = [u_1, v_1, u_3, v_3]$$

$$[k_g]_2 = \frac{EA}{4L} \begin{bmatrix} 1 & \sqrt{3} & -1 & -\sqrt{3} \\ \sqrt{3} & 3 & -\sqrt{3} & -3 \\ -1 & -\sqrt{3} & 1 & \sqrt{3} \\ \sqrt{3} & -3 & \sqrt{3} & 3 \end{bmatrix}$$

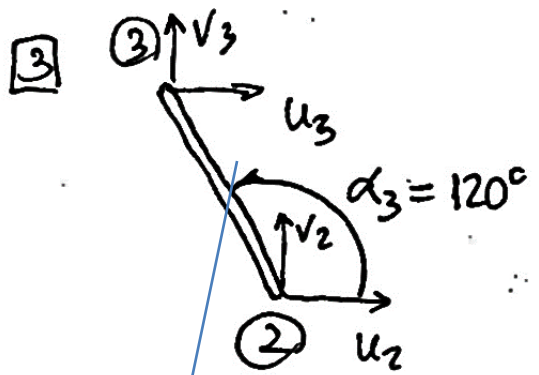


$$[K_g]_2^* = \frac{EA}{4L}$$

$$\begin{bmatrix} 1 & \sqrt{3} & 0 & 0 & -1 & -\sqrt{3} & 0 & 0 \\ \sqrt{3} & 3 & 0 & 0 & -\sqrt{3} & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -\sqrt{3} & 0 & 0 & 1 & \sqrt{3} & 0 & 0 \\ -\sqrt{3} & -3 & 0 & 0 & \sqrt{3} & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[k_s]_e = \frac{EA}{l_e} \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

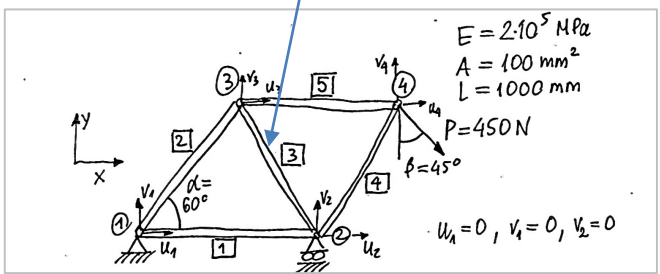
$s = \sin \alpha, c = \cos \alpha$



$$C_3 = -\frac{1}{2}, \quad S_3 = \frac{\sqrt{3}}{2}, \quad [q_3]_3 = [u_2, v_2, u_3, v_3]$$

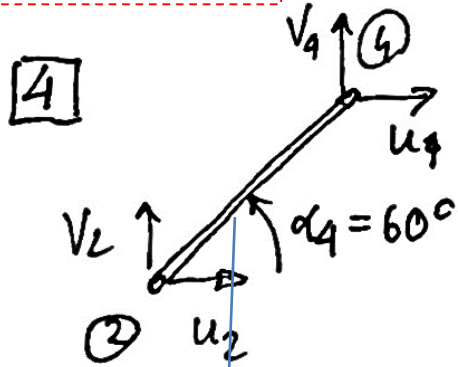
$$[k_g]_3 = \frac{EA}{4L} \begin{bmatrix} 1 & -\sqrt{3} & -1 & \sqrt{3} \\ -\sqrt{3} & 3 & \sqrt{3} & -3 \\ -1 & \sqrt{3} & 1 & -\sqrt{3} \\ \sqrt{3} & -3 & -\sqrt{3} & 3 \end{bmatrix}$$

$$[k_g]_3^* = \frac{EA}{4L} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\sqrt{3} & -1 & \sqrt{3} & 0 & 0 \\ 0 & 0 & -\sqrt{3} & 3 & \sqrt{3} & -3 & 0 & 0 \\ 0 & 0 & -1 & \sqrt{3} & 1 & -\sqrt{3} & 0 & 0 \\ 0 & 0 & \sqrt{3} & -3 & -\sqrt{3} & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



$$[k_s]_e = \frac{EA}{l_e} \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

$s = \sin \alpha, c = \cos \alpha$



$$c_4 = \frac{1}{2}, s_4 = \frac{\sqrt{3}}{2}, L_{[g]_4} = [u_2, v_2, u_4, v_4]$$

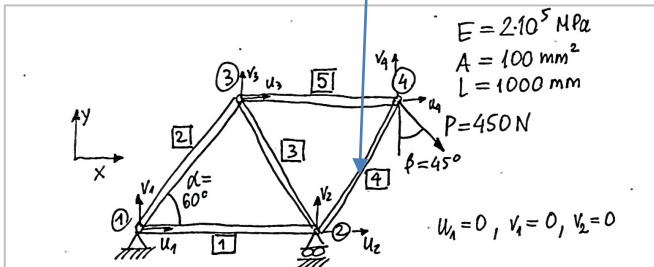
$$[k_g]_4 = \frac{EA}{4l} \begin{bmatrix} 1 & \sqrt{3} & -1 & -\sqrt{3} \\ \sqrt{3} & 3 & -\sqrt{3} & -3 \\ -1 & -\sqrt{3} & 1 & \sqrt{3} \\ -\sqrt{3} & -3 & \sqrt{3} & 3 \end{bmatrix}$$

4×4

$$[k_g]_4^* = \frac{EA}{4l}$$

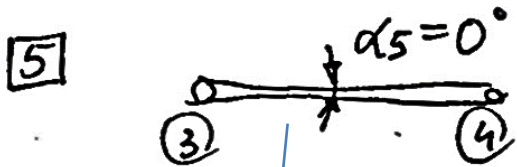
8×8

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \sqrt{3} & 0 & 0 & -1 & -\sqrt{3} \\ 0 & 0 & \sqrt{3} & 3 & 0 & 0 & -\sqrt{3} & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -\sqrt{3} & 0 & 0 & 1 & \sqrt{3} \\ 0 & 0 & -\sqrt{3} & -3 & 0 & 0 & \sqrt{3} & 3 \end{bmatrix}$$



$$[k_s]_e = \frac{EA}{l_e} \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

$s = \sin \alpha, c = \cos \alpha$



$$c_5 = 1, s_5 = 0, [q]_5 = [u_3, v_3, u_4, v_4]$$

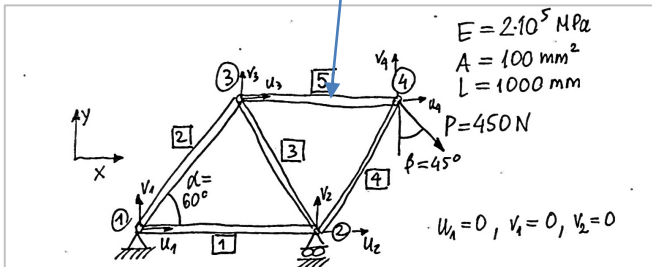
$$[k_g]_5 = \frac{EA}{4L} \begin{bmatrix} 4 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 \\ -4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

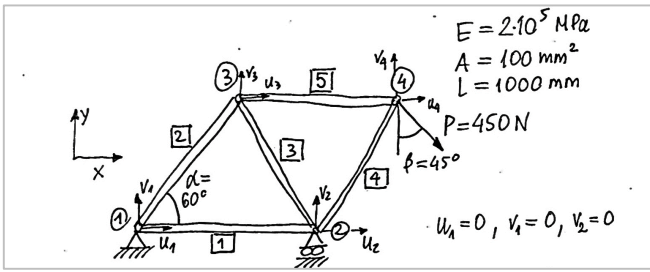
4x4

$$[k_g]_5^* = \frac{EA}{4L}$$

8x8

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



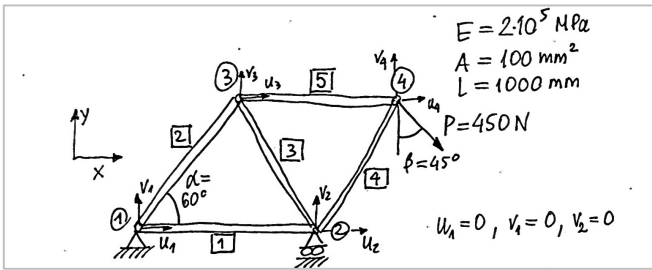


$$[K]_{8 \times 8} = \sum_{e=1}^5 [K_e]_e^* = \frac{EA}{4L}$$

$$\begin{bmatrix} 5\sqrt{3} & -4 & 0 & -1 & -\sqrt{3} & 0 & 0 & 0 \\ \sqrt{3} & 3 & 0 & 0 & -\sqrt{3} & -3 & 0 & 0 \\ -4 & 0 & 6 & 0 & -1 & \sqrt{3} & -1 & -\sqrt{3} \\ 0 & 0 & 0 & 6 & \sqrt{3} & -3 & -\sqrt{3} & -3 \\ -1 & -\sqrt{3} & -1 & \sqrt{3} & 6 & 0 & -4 & 0 \\ -\sqrt{3} & -3 & \sqrt{3} & -3 & 0 & 6 & 0 & 0 \\ 0 & 0 & -1 & -\sqrt{3} & -4 & 0 & 5 & \sqrt{3} \\ 0 & 0 & -\sqrt{3} & -3 & 0 & 0 & \sqrt{3} & 3 \end{bmatrix}$$

$$[K]_{8 \times 8} \cdot \{q\}_{8 \times 1} = \{F\}_{8 \times 1}$$

+ boundary conditions : $u_1 = 0, v_1 = 0, v_2 = 0$



$$[K] \cdot \{q\} = \{F\}$$

$5 \times 5 \quad \quad \quad 5 \times 1 \quad \quad \quad 5 \times 1$

$$\frac{EA}{4L} \begin{bmatrix} 6 & -1 & \sqrt{3} & -1 & -\sqrt{3} \\ -1 & 6 & 0 & -4 & 0 \\ \sqrt{3} & 0 & 6 & 0 & 0 \\ -1 & -4 & 0 & 5 & \sqrt{3} \\ -\sqrt{3} & 0 & 0 & \sqrt{3} & 3 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \frac{\sqrt{2}}{2} P \\ -\frac{\sqrt{2}}{2} P \end{Bmatrix}$$

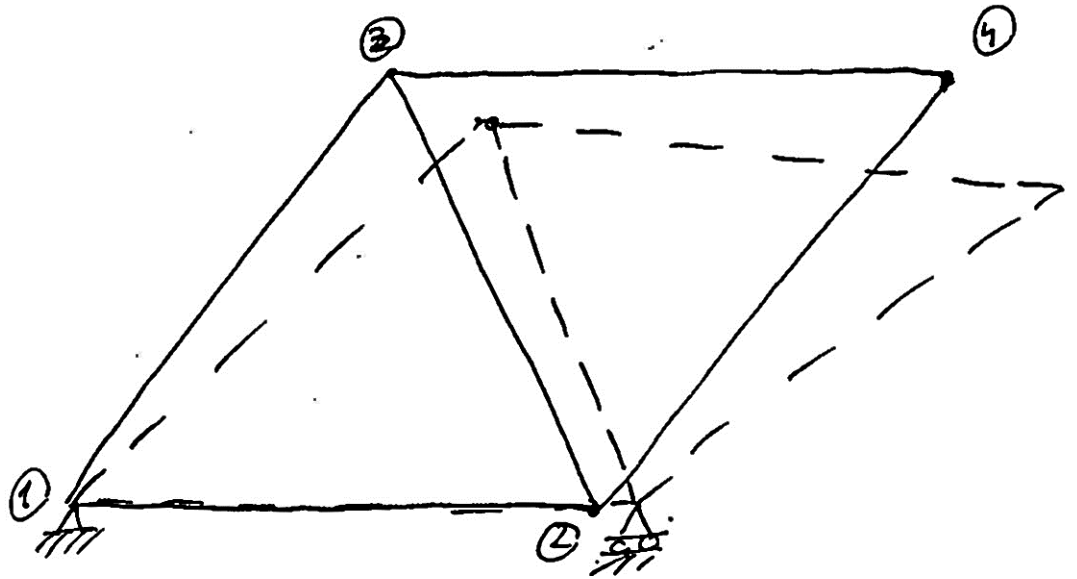
$$u_2 = 0.3362 \cdot 10^{-2} \text{ mm}$$

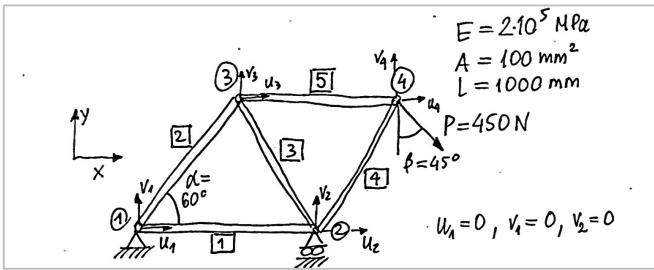
$$u_3 = 5.1872 \cdot 10^{-2} \text{ mm}$$

$$v_3 = -0.09706 \cdot 10^{-2} \text{ mm}$$

$$u_4 = 7.6968 \cdot 10^{-2} \text{ mm}$$

$$v_4 = -6.3705 \cdot 10^{-2} \text{ mm}$$





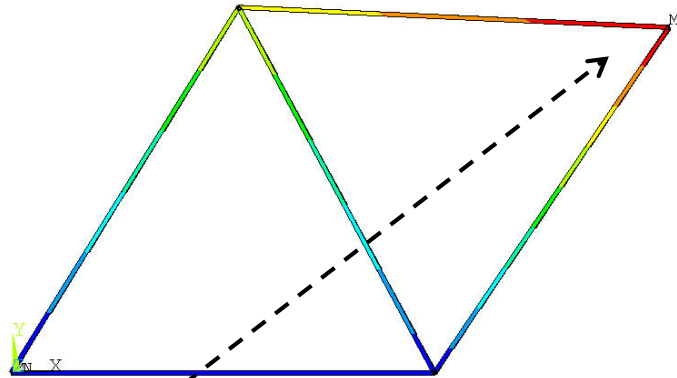
$$u_2 = 0.3362 \cdot 10^{-2} \text{ mm}$$

$$u_3 = 5.1872 \cdot 10^{-2} \text{ mm}$$

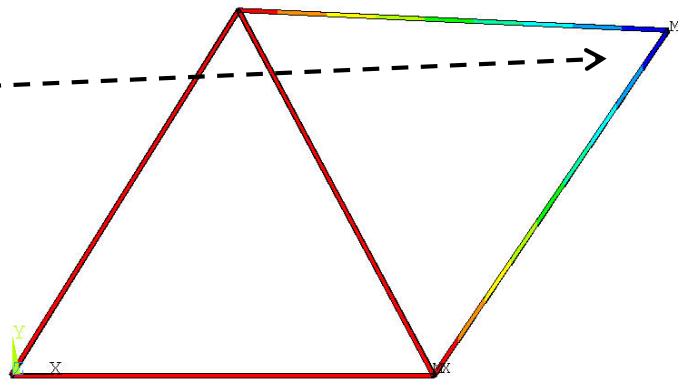
$$v_3 = -0.09706 \cdot 10^{-2} \text{ mm}$$

$$u_4 = 7.6968 \cdot 10^{-2} \text{ mm}$$

$$v_4 = -6.3705 \cdot 10^{-2} \text{ mm}$$

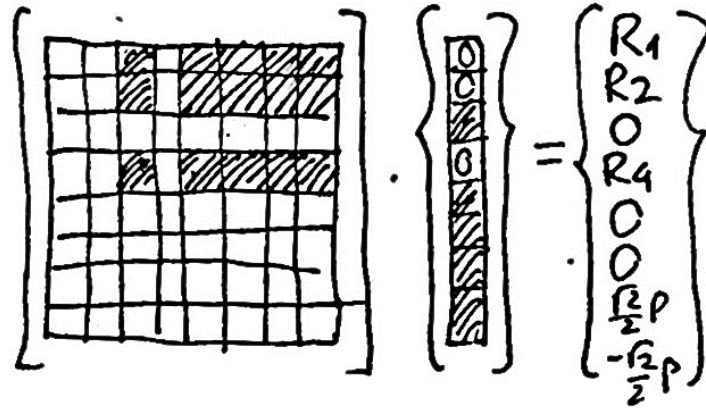


UX (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX = .100156
 SMN = -.114E-03
 SMX = .077281
 -.114E-03
 .008486
 .017085
 .025685
 .034284
 .042884
 .051483
 .060082
 .068682
 .077281

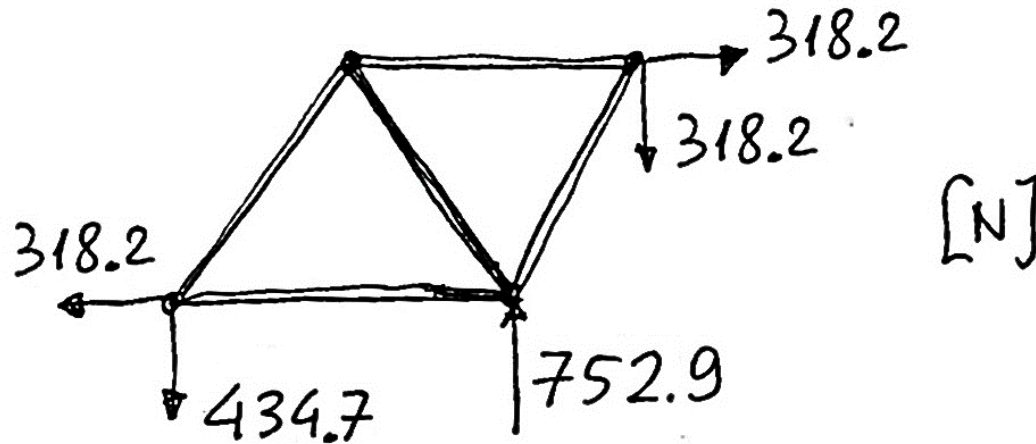


UY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX = .100156
 SMN = -.064123
 SMX = .414E-03
 -.064123
 -.056952
 -.049782
 -.042611
 -.03544
 -.028269
 -.021098
 -.013928
 -.006757
 .414E-03

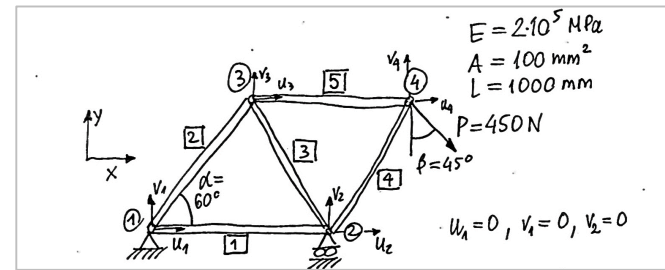
Reakcje



$$\begin{aligned} R_1 &= -318.2 \text{ N} \\ \Rightarrow R_2 &= -434.7 \text{ N} \\ R_4 &= 752.9 \text{ N} \end{aligned}$$



Naprężenia i siły wewnętrzne



$$\boxed{1} \quad \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_1 = \begin{bmatrix} T_t \\ 1 \end{bmatrix}_1 \cdot \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_1 = \begin{bmatrix} c_1 & s_1 & 0 & 0 \\ 0 & 0 & c_1 & s_1 \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}_1 = \begin{Bmatrix} 0 \\ 0.33622 \cdot 10^{-2} \end{Bmatrix}$$

$2 \times 1 \quad \quad 2 \times 4 \quad \quad 4 \times 1 \quad \quad (2 \times 4) \quad \quad (4 \times 1)$

$$\sigma_1 = \frac{E}{L} (q_2 - q_1)_1 = 0.67 \text{ MPa}, \quad N_1 = \sigma_1 \cdot A = 672.4 \text{ N}$$

$$\boxed{2} \quad \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_2 = \begin{bmatrix} T_t \\ 1 \end{bmatrix}_2 \cdot \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_2 = \begin{bmatrix} c_2 & s_2 & 0 & 0 \\ 0 & 0 & c_2 & s_2 \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ v_1 \\ u_3 \\ v_3 \end{Bmatrix}_2 = \begin{Bmatrix} 0 \\ 2.5096 \cdot 10^{-2} \end{Bmatrix}$$

$2 \times 1 \quad \quad 2 \times 4 \quad \quad 4 \times 1 \quad \quad (2 \times 4) \quad \quad (4 \times 1)$

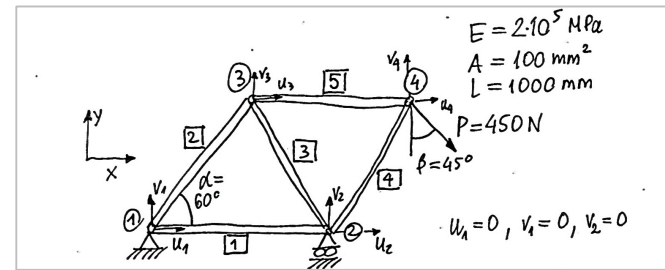
$$\sigma_2 = \frac{E}{L} (q_2 - q_1)_2 = 5.02 \text{ MPa}, \quad N_2 = \sigma_2 \cdot A = 502 \text{ N}$$

$$\boxed{3} \quad \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_3 = \begin{bmatrix} T_t \\ 1 \end{bmatrix}_3 \cdot \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_3 = \begin{bmatrix} c_3 & s_3 & 0 & 0 \\ 0 & 0 & c_3 & s_3 \end{bmatrix} \cdot \begin{Bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}_3 = \begin{Bmatrix} -0.16811 \cdot 10^{-2} \\ -2.67766 \cdot 10^{-2} \end{Bmatrix}$$

$2 \times 1 \quad \quad 2 \times 4 \quad \quad 4 \times 1 \quad \quad (2 \times 4) \quad \quad (4 \times 1)$

$$\sigma_3 = \frac{E}{L} (q_2 - q_1)_3 = -5.02 \text{ MPa}, \quad N_3 = \sigma_3 \cdot A = -502 \text{ N}$$

(possible buckling)



$$\boxed{4} \quad \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_4 = [T_t]_4 \cdot \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_4 = \begin{bmatrix} c_4 & s_4 & 0 & 0 \\ 0 & 0 & c_4 & s_4 \end{bmatrix} \cdot \begin{Bmatrix} u_2 \\ v_2 \\ u_4 \\ v_4 \end{Bmatrix} = \begin{Bmatrix} 0.16811 \cdot 10^{-2} \\ -1.66901 \cdot 10^{-2} \end{Bmatrix}$$

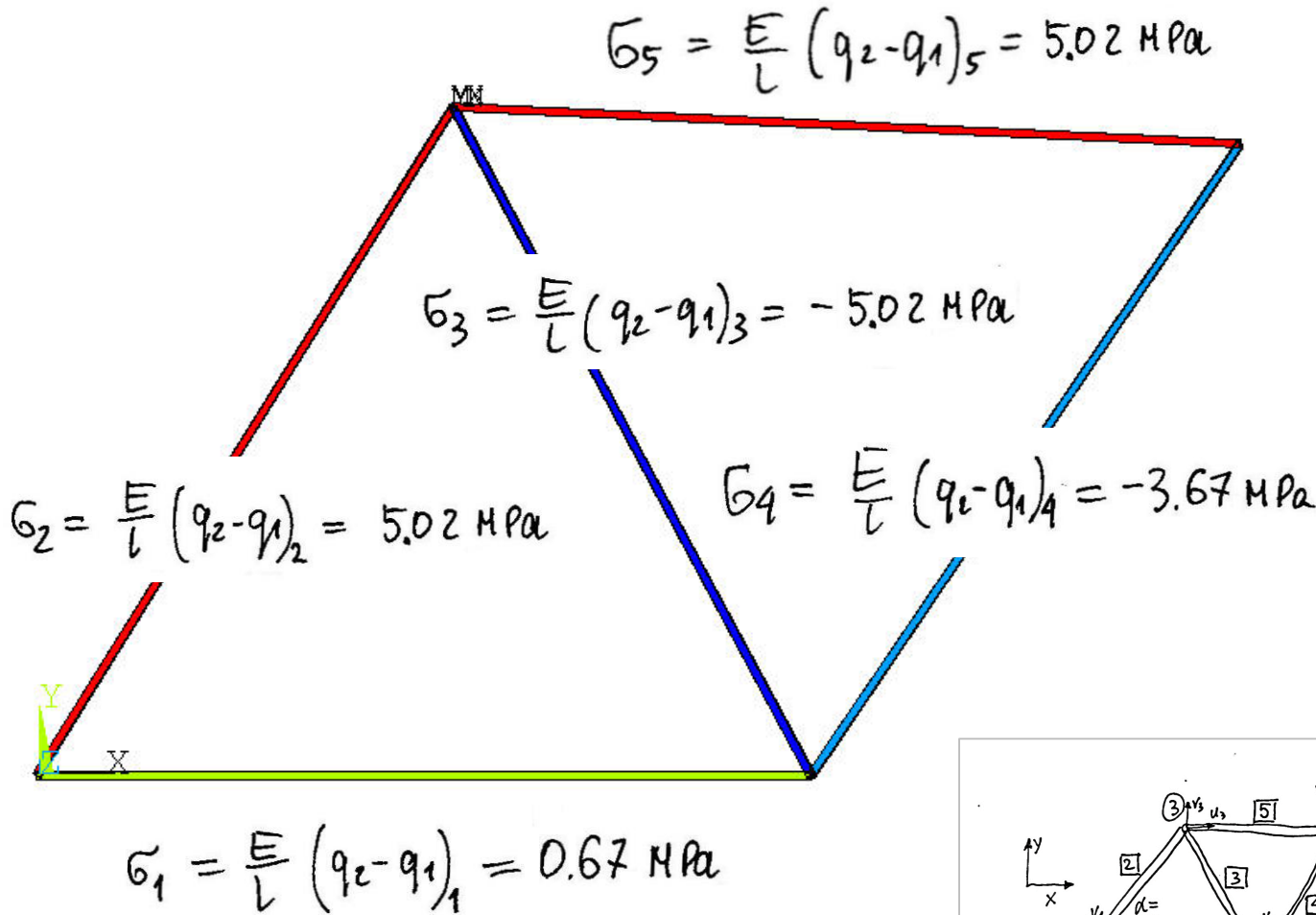
$$\sigma_4 = \frac{E}{L} (q_2 - q_1)_4 = -3.67 \text{ MPa}, \quad N_4 = \sigma_4 A = -367 \text{ N} \quad \text{②} \nearrow \text{④}$$

(possible buckling)

$$\boxed{5} \quad \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_5 = [T_t]_5 \cdot \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_5 = \begin{bmatrix} c_5 & s_5 & 0 & 0 \\ 0 & 0 & c_5 & s_5 \end{bmatrix} \begin{Bmatrix} u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix} = \begin{Bmatrix} 5.18721 \cdot 10^{-2} \\ 7.6968 \cdot 10^{-2} \end{Bmatrix}$$

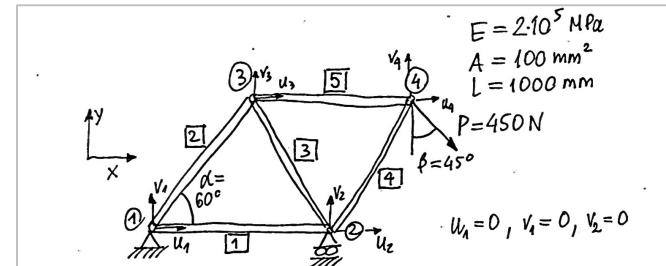
$$\sigma_5 = \frac{E}{L} (q_2 - q_1)_5 = 5.02 \text{ MPa}, \quad N_5 = 502 \text{ N}$$

Naprężenia [MPa]

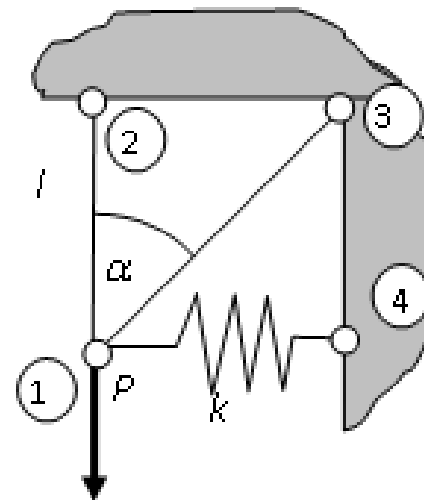
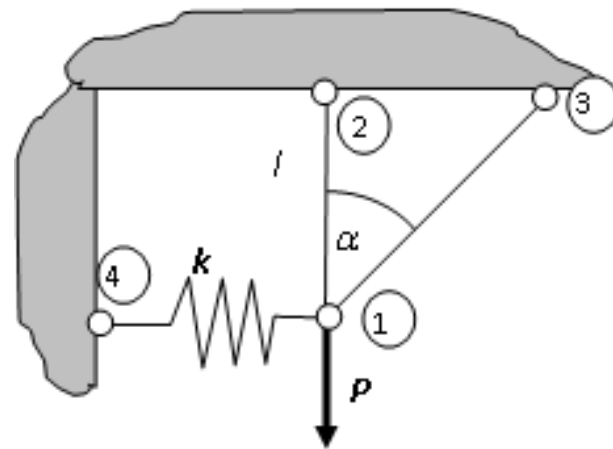
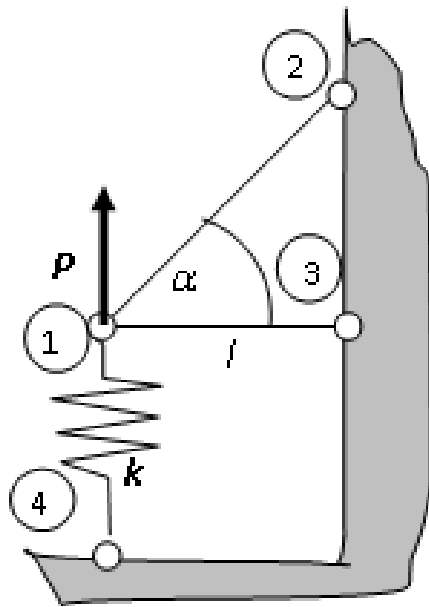


SX (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX = .100156
 SMN = -5.0191
 SMX = 5.0191

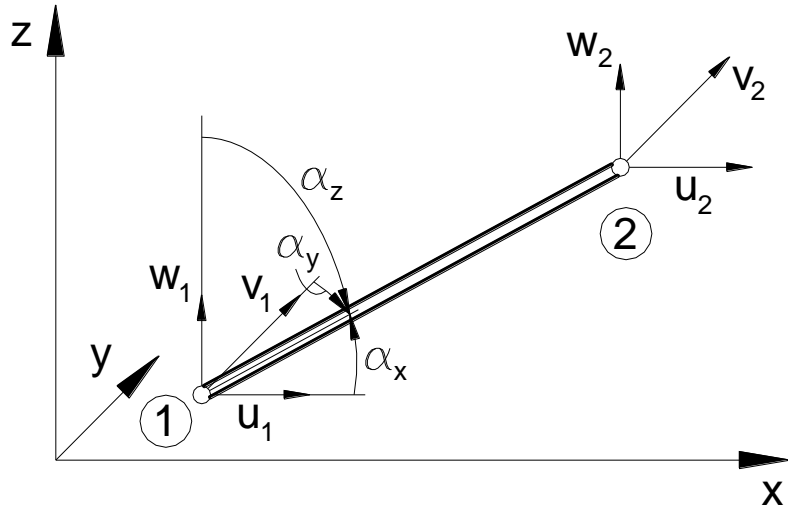
Blue	-5.0191
Light Blue	-3.90374
Cyan	-2.78839
Light Green	-1.67303
Green	-.557678
Yellow-Green	.557678
Yellow	1.67303
Orange	2.78839
Red-Orange	3.90374
Red	5.0191



Przykłady zadań

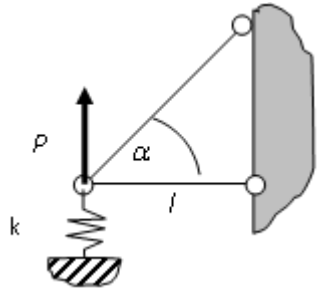


Element skończony pręta kratownicy 3D



$$\{q\}_e = \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \end{Bmatrix} \quad \text{Globalny wektor parametrów węzłowych}$$

Globalna macierz sztywności pręta kratownicy:



$$[k^g]_e = \frac{EA}{l_e} \begin{bmatrix} c_x^2 & c_x c_y & c_x c_z & -c_x^2 & -c_x c_y & -c_x c_z \\ c_x c_y & c_y^2 & c_y c_z & -c_x c_y & -c_y^2 & -c_y c_z \\ c_x c_z & c_y c_z & c_z^2 & -c_x c_z & -c_y c_z & -c_z^2 \\ -c_x^2 & -c_x c_y & -c_x c_z & c_x^2 & c_x c_y & c_x c_z \\ -c_x c_y & -c_y^2 & -c_y c_z & c_x c_y & c_y^2 & c_y c_z \\ -c_x c_z & -c_y c_z & -c_z^2 & c_x c_z & c_y c_z & c_z^2 \end{bmatrix}$$

$$c_x = \cos \alpha_x$$

$$c_y = \cos \alpha_y$$

$$c_z = \cos \alpha_z$$